

A Logical Scheme and Paraconsistent Topological Separation in Byzantium: Inter-Trinitarian Relations according to Hieromonk Hierotheos and Joseph Bryennios¹

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Ὅρᾱς χορεῖαν ξένην ἣν οὐδέποτε εἶδες ;
*Do you see the strange round dance that you have never seen before?*²

1. Introduction

Hieromonk Hierotheos who *floruit* in the second half of the thirteenth century is not a famous theologian – but he, nevertheless, became the hidden key figure of the late Byzantine Triadology. It was Joseph Bryennios (*ca.* 1350–1431/8) who was responsible for this fact, without, however, rescuing his name from oblivion. The preserved Hierotheos's works were published not long ago³ but remain understudied.

In the present study, I will address Joseph Bryennios as the porte-parole of Hierotheos, even though Bryennios himself was, probably, the most important theologian of his epoch and provided theological background to Mark of Ephesus and Gennadios Scholarios.⁴

Gabriele Patacsi noticed, as early as in 1973, that it was hieromonk Hierotheos whom Bryennios quoted under the name of the homonymous legendary first-century bishop of Athens and the teacher of Dionysius the Areopagite according to the *Corpus Areopagiticum*; Patacsi pointed out as well a manuscript of a hieromonk Hierotheos's work reattributed to the same Hierotheos.⁵ Thus, Bryennios was following a tradition of pseudepigraphy already established before him – and made “canonical” for later generations by himself.

During his lifetime, hieromonk Hierotheos was a very controversial theologian, maybe the most controversial one in the camp of the adversaries of the Union of Lyon (1274–1283). He was attacked not only from the side of the *latinophrones* (partisans of the union with Rome) but also from the side of conservative anti-Latin Greek theologians, who managed to establish their preponderance in the official Church of Constantinople after having deposed, in 1289, Patriarch of Constantinople Gregory of Cyprus. Patriarch Gregory was a theologian, who belonged to the same party as Hierotheos.⁶ In one of these “dark decades” for Byzantine theology, hieromonk Hierotheos died (*ca.* the year 1300). Unlike their conservative opponents, both Gregory of Cyprus and hieromonk Hierotheos were not limiting themselves to criticize the Latin *Filioque* but both provided its interpretation compatible with the Byzantine Orthodoxy. Their ideas on the mutual relationship between the Son and the Spirit in the Trinity became recalled in the 1330s, at the latest, by the Palamite theologians including Gregory

¹ The present study is a part of a larger project Nr 16-18-10202, *History of the Logical and Philosophical Ideas in Byzantine Philosophy and Theology*, implemented with a financial support of the Russian Science Foundation. I am very grateful to all those who supported my work in different ways, especially Dmitry Biriukov, Alex Simonov, Dmitry Makarov, and Olga Mitrenina. I am also very grateful to the participants of the round table “Paraconsistent Logics and Their Place in Theology” organised in the Institute of Philosophy of the Siberian Branch of the Russian Academy of Sciences, Novosibirsk, on 12 October 2017, and especially to Vitaly V. Tselishchev for a fruitful discussion of main logical ideas of this paper. Any possible shortcomings and errors are, of course, of my own.

² For the source and the meaning of this sentence, see below, section 4.

³ The comprehensive review of Hierotheos's legacy is the paper by Gabriele Patacsi, ‘Le hiéromoine Hiérothée, théologien du Saint-Esprit’, *Κληρονομία* 13 (1981) 299–330. The preserved corpus of works is published by Nikolaos Ioannides: N. X. Ἰωαννίδης, *Ὁ Ἱερομόναχος Ἱερόθεος (II' αἰ.) καὶ τὸ ἀνέκδοτο συγγραφικὸ ἔργο του*, Athens: Κυριακίδη, 2003; 2nd ed. 2007.

⁴ Cf. Basil Lourié, ‘Joseph Bryennios and Theological Testament of Byzantium’ (forthcoming).

⁵ Gabriele Patacsi, ‘Joseph Bryennios et les discussions sur un concile d’union (1414–1431)’, *Κληρονομία* 5 (1973) 73–96 (pp. 81–2); cf. Patacsi, ‘Hierothée’, pp. 327–28.

⁶ As an introduction to the corresponding theological discussions, one can consult Aristeides Papadakis, *Crisis in Byzantium: The Filioque Controversy in the Patriarchate of Gregory II of Cyprus (1283–1289)*, rev. edn (Crestwood, NY: St. Vladimir's Seminary Press, 1997), together with *La vie et l'œuvre théologique de Georges/Grégoire II de Chypre (1241–1290) patriarche de Constantinople*, ed. by Jean-Claude Larchet (Paris: Cerf, 2012).

Palamas (1296–1357) himself. In this way, these ideas became the new mainstream, thus paving the road to ascription of Hierotheos's works to the apostolic figure of Hierotheos of Athens.⁷ Nevertheless, Hierotheos's reasoning was far more formal than even his sympathisers were familiarised with. This fact seems to be the most affecting the destiny of Hierotheos's literary work. His treatises became relatively rarely copied – and, no doubt, would have been fallen into oblivion definitively, if Bryennios did not become fascinated by them. Then, Bryennios played the part of a promoter of Hierotheos's theology – of course, under the guise of that of Hierotheos of Athens. Below, I will trace Hierotheos's thought as it was made accessible by Bryennios, thus postponing a direct study of Hierotheos's works until a separate paper. Understanding Bryennios's perception of them is a task that has an importance of its own.

2. Bryennios's Adaptation of Hierotheos's Logical Scheme

The most striking feature of Hierotheos's theological reasoning was his predilection to graphical schemes. Before him, graphical illustrations in theological treatises were in order as well, but he made a decisive step forward: he started to draw logical propositions thus making the graphical objects that we call now logical schemes.

The logical schemes are not simply visualisations of thoughts but a tool of logical calculation. According to the present consensus among the historians of logic, the first logical schemes were authored by Leibniz, whereas their general acceptance goes back to Leonard Euler, 1763.⁸

I think, however, that some rudimentary logical schemes appeared in Byzantium even before hieromonk Hierotheos,⁹ whereas Hierotheos created some very elaborated ones. On Fig. 1, we reproduce a Hierotheos's drawing containing two logical schemes serving to a unique theological purpose. This graphical chart has been repeated by Joseph Bryennios, and we reproduce it according to the etching in the 1768 edition of the latter's works.¹⁰ This edition is a faithful reproduction of the original Bryennios's and, through him, Hierotheos's drawing, despite some baroque decorative elements added according to the taste of the epoch.

PICTURE (600 dpi) FROM THE ATTACHED FILE NAMED "0001". The picture must occupy the whole page.

Fig. 1. Orthodox Triadology according to hieromonk Hierotheos and Joseph Bryennios.

We have no room here to discuss the immediate context of theological polemics of either Hierotheos or Bryennios.¹¹ It is sufficient to note that the scheme belongs to the key topic of both Hierotheos's

⁷ As a partial introduction to the triadological problematics in Byzantium between the 1250s and the early 15th century, see Basil Lourié, 'Nicephorus Blemmydes on the Holy Trinity and the Paraconsistent Notion of Numbers: A Logical Analysis of a Byzantine Approach to the *Filioque*', *Studia Humana* 5 (2016) 40–54.

⁸ Gailand Mac Queen, *The Logic Diagram*. MA Thesis (Hamilton: McMaster University, 1967) (never published but influential historical work); Amirouche Moktefi, Sun-Joo Shin, 'A History of Logic Diagrams', in *Logic: A History of Its Central Concepts*, ed. by Dov M. Gabbay, Francis J. Pelletier and John Woods, *Handbook of the History of Logic*, vol. 11 (Amsterdam: Elsevier, 2012), pp. 611–82.

⁹ Dmitry Afinogenov drew my attention to a rudimentary logical scheme in a work of Patriarch Photios written between 875 and 877 (*Amphilochia*, 78; Leendert G. Westerink, *Photii Patriarchae Constantinopolitani Epistulae et Amphilochia*. Vol. 5. Bibliotheca scriptorum graecorum et romanorum Teubneriana (Leipzig: Teubner, 1986), pp. 101–08 (p. 103), where a simple logical scheme is used for Christological propositions.

¹⁰ Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, *Τὰ εἰρηθέεντα*, [ed. by Eugenios Voulgaris], τ. Α' (Leipzig: Βρεϊτκόπφ, 1768) [thereafter referred to by the pages in the text]. No modern edition is available, although the 1768 one has been reprinted, using a modern Greek font, by the publishing house of V. Rigopoulos [B. Ρηγούπουλου], Thessalonica, in 1991; in this edition, however, the graphical charts are simplified to the extent of becoming almost useless for us now. We are interested only in Chart Γ', on the second separate sheet, out of pagination. Charts Α' and Β', also borrowed by Bryennios in Hierotheos, are reproduced together on the first separate sheet; they will be not dealt with here, because they are simply illustrations of, respectively, Byzantine and Latin Triadologies without being tools of visual reasoning (logical schemes).

¹¹ Meanwhile, one can consult Lourié, 'Nicephorus Blemmydes'; cf. Lourié, 'Joseph Bryennios'.

and Bryennios's (and, after him, Mark's of Ephesus and the Council's of Florence, 1438–1439) triadological polemics, the problem of (in)existence of a natural order between the hypostases of the Trinity.

The relevant triadological ideas of these Greek theologians could be summarised as following. The existence of the order between the three hypostases in revelation and other phenomena *ad extra* is out of doubt and, therefore, out of discussion. Some kind of natural order would have been implied, however, by the *Filioque*. The radical claim of Orthodox polemicists, especially after Hierotheos in the 1270s, consisted in denying any kind of natural order between the three hypostases except the causal priority of the Father. The mutual relations between the Son and the Spirit, without being causal, must have been perfectly symmetrical – in the way that the Spirit is the Spirit of the Son in the same sense and in the same extent as the Son is the Son of the Spirit. The *Filioque* would have been tolerable as a correct expression of faith only together with the *Spirituque*, but without any causation overtones (such as those appeared in the twentieth-century *Spirituque* doctrines). If we take causation apart, even the Father ceases to be the first among the three: the non-causal inter-trinitarian relations are perfectly symmetrical. Therefore, each one of the three hypostases could be called the first or the second or the third.

Such an approach inevitably resulted into logical difficulties affecting the very sense of “three” and the very notion of number when applied to the Trinity. Hierotheos and Bryennios were not the first who dared to discuss them overtly – the first were Gregory of Nazianzus and Evagrius Ponticus in the second half of the fourth century¹² – but their approach turned out to be the most formal.

3. The Logical Sondergut of Byzantine Triadology

Even at first glance our logical scheme (Fig. 1) reveals a deep logical problem: it describes mutual relations between six elements, which may seem twice as large as it should be required if we have only one Trinity and not two.

The duplicate names for each hypostasis, namely, Πατήρ (‘Father’) and Προβολεύς (‘Projector’), Υἱός (‘Son’) and Λόγος (‘Logos, Word’), and Πνεῦμα (‘Spirit’) and Πρόβλημα (‘Projection’), are called by Bryennios¹³ different “names” (ὀνόματα) applied to the same hypostasis, but, nevertheless, they imply some ontological difference – within an object, a hypostasis, that is still identical with itself. This ontological difference is of such a great importance that it is the only responsible for the inter-trinitarian mutual relations, that is, for excluding the *Filioque* and granting the perfect symmetry between the three. This difference between the “names” within each of three combinations does not belong to the human intellect but is a feature of the divinity.

In logical terms, this means that the law of identity is broken in a non-extensional way: *A* is still identical to *A* but there are two different *As* identical to each other. The relation of identity continues to be reflexive (because we still have $A = A$), as it is in any consistent logic, but becomes “more than reflexive” or, I would term it, super-reflexive, in the sense that there are two different *As*. The term “super-reflexive” is constructed after the pattern of the term “non-reflexive” that characterises the relation of identity between the non-individuals, such as the quantum objects.¹⁴ For the non-individuals, there is no reflexivity in identity, $A \neq A$. Then, for the super-reflexive identity, we would have the conjunction $A = A \wedge A \neq B \wedge B = B \wedge A = B$, or, in a generalised form, $(a_i = a_i \neq a_j) \wedge (a_i = a_j)$, $i \neq j$, $i \in \mathbb{N}^+$, $j \in \mathbb{N}^+$. The so-called non-reflexive logic deals with contrary oppositions and is,

¹² Basil Lourié, ‘What Means “Tri-” in “Trinity”? An Eastern Patristic Approach to the “Quasi-Ordinals”’, *Journal of Applied Logic* (forthcoming).

¹³ The references to Bryennios will be limited to a unique treatise where he elaborates on the present graphical chart, his theological testament called *The Hortatory Sermon on the Unity of the Churches* (Λόγος συμβουλευτικός περὶ τῆς ἐνώσεως τῶν ἐκκλησιῶν), 1422; Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, τ. Α΄, pp. 469–500, esp. 487–500.

¹⁴ See especially: Steven French, Décio Krause, *Identity in Physics: A historical, philosophical and formal analysis* (Oxford: Oxford University Press, 2006); Jonas R. B. Arenhart, Décio Krause, ‘Classical logic or non-reflexive logic? A case of semantic underdetermination’, *Revista Portuguesa de Filosofia* 68 (2012) 73–86; Newton C. A. da Costa, Christian de Ronde, ‘Non-reflexive Logical Foundation for Quantum Mechanics’, *Foundations of Physics* 44 (2014) 1369–80.

therefore, paraconsistent, whereas the super-reflexive logic deals with subcontrary oppositions and is, therefore, paraconsistent.¹⁵

For understanding of the logical side of this Triadology, we need to realise what happens to the numbers. The non-reflexive logic elaborated for the quantum objects is suitable for counting the non-individuals. It is also called “Schrödinger’s logic” referring to his comparison between counting the non-individual electrons and the non-individual dollars in bank account. These dollars are not discernible but countable. The corresponding numbers are called “quasi-cardinals”, because they are similar to the cardinals but not derived from a row of the ordinal numbers. The ordinal numbers for the non-individuals are impossible because the non-individuals do not form the permutations¹⁶ and, therefore, do not form the ordered pairs and an ordered row.

The true natural numbers, in their set-theoretical representation, are supposed to meet the axiom of extensionality:

$$\forall a \forall b [\forall x (x \in a \leftrightarrow x \in b) \rightarrow a = b]$$

In the words: if, for any sets a and b , it is true that, for any set x , x is a member of a if and only if it is a member of b , then the sets a and b are identical. In other words: if any two sets have all their members identical, these sets themselves are identical.

The quasi-cardinals were defined for a reformulation of the quantum theory in order to allow the counting of the non-individuals (such as the quantum objects) without referring to the set theories implying the sets which elements are crisp individuals. For the non-individuals, the permutations are impossible and, therefore, the ordered pairs are impossible either. Nevertheless, they are still countable – in the same way as, according to the Erwin Schrödinger’s comparison, the dollars in bank account are indistinguishable but not identical and, therefore, countable.

In Triadology, the elements of paraconsistent oppositions are distinguishable and not identical and, therefore, *a fortiori* countable. The permutations and the ordered pairs are permitted. This means, that there could be some analogy with an ordered row – even though the row of natural numbers itself is, of course, not permitted. This is why I have called the numbers corresponding to our case “quasi-ordinal” ones.¹⁷

The standard definition of the ordered pair as it was formulated by Kazimierz Kuratowski in 1921 is the following:

$$(a, b) := \{\{a\}, \{a, b\}\}$$

In Bryennios’s analysis, some kind of order is appropriate only as an expression of the $\mu\upsilon\nu\alpha\rho\chi\acute{\iota}\alpha$ of the Father, that is, the principle that the Father is the only “beginning” in the Trinity. If we try to construct a corresponding ordered pair, we will have two second elements instead of one; I call such pairs pseudo-ordered. This pair would be a paraconsistent conjunction as following:

¹⁵ According to the definition introduced by the Peruvian logician Francisco Miró Quesada Cantuarias in 1956 and then made standard by Newton C. A. da Costa, paraconsistent contradiction, if allowed, would break the principle of excluded middle, whereas the paraconsistent contradiction, the principle of non-contradiction (and, therefore, the principle of identity as well, the latter being equivalent to the former). If A and B form a paraconsistent opposition, the classical logic would allow $X = A$ and $X = B$ but not $X = \text{not-}A$ and $X = \text{not-}B$; if they form a paraconsistent opposition, the classical logic would allow $X = \text{not-}A$ and $X = \text{not-}B$ but not $X = A$ and $X = B$. Schrödinger’s Cat (an example of quantum superposition) belongs to the realm of paraconsistent logics, whereas the Byzantine triadological models to that of paraconsistent ones.

¹⁶ The terms of combinatorics “combination” and “permutation” differ in the way that the combinations do not imply order (all their elements could occupy any position in the group), whereas the permutations are the ordered combinations.

¹⁷ For a discussion of this problem within a large historical retrospective of Christian Triadology, see Lourié, ‘Nicephorus Blemmydes’ and *idem* = Василий Лурье, ‘Понятие числа в триадологии восточной патристики [The Notion of Number in the Triadology of Eastern Patristics]’, *Esse* 1.1 (2016) <http://esse-journal.ru/?p=1713>.

$$(1) \quad \left(a, \bigwedge_{n-1} b_{n-1} \right) = \bigwedge_{n-1} \{ \{a\}, \{a, b_{n-1}\} \}$$

where n is the total number of the elements in the system (for the Trinity, $n = 3$).

This is applied to the causal relations in the Holy Trinity only. Otherwise, in the non-causal respects, however, even the Father is not “the first”.

The appropriate generalisation of the definition (1) results into the definition (2) of the pseudo-ordered pair:

$$(2) \quad \bigwedge_n (a_i, a_j) = \bigwedge_n \{ \{a_i\}, \{a_i, a_j\} \}$$

where $0 < i \neq j \leq n, n \in \mathbb{N}^+$.

This means that all permutations from 3 by 2 are implied simultaneously. Their number is 6, according to the formula of combinatorics (after the substitution $n = 3$ and $m = 2$):

$$P_n^m = \frac{n!}{(n-m)!}$$

Now we can understand better an unknown pro-Latin Greek author who, in the 1440s, wrote against Mark of Ephesus (who was elaborating on Bryennios’s ideas), that the Latins acknowledge only that kind of order in the Trinity that makes possible counting to three:¹⁸

Τὴν ἐκβαλλομένην τοίνυν ὑπὸ τῶν ἀγίων τάξιν ἀπὸ Θεοῦ καὶ οἱ Λατῖνοι συνεκβάλλουσι δῆπου· τὴν δὲ τάξιν, ἧς ἄνευ ἀδύνατον καὶ Τριάδα λέγεσθαι τὴν μακαρίαν Τριάδα <...> οὐ τολμῶσιν ἀρνεῖσθαι...

The order in God that the saints have rejected, the Latins, of course, reject as well. However, the order without which it is impossible even to call “Trinity” the blessed Trinity <...> they do not dare to withdraw...

Hierotheos and Bryennios, like their patristic predecessors, were preoccupied exactly with this topic: to explain that the Holy Trinity is not a triad in the sense of the natural cardinal number three and not a consistent number at all. This was the *Sondergut* of the Byzantine Triadology, which now became articulated in a new and the most formal way.

4. The Logical Scheme and Its Syllogisms

Bryennios provided a – incomplete, as we will see below (section 6) – description of the chart and introduced alphabetic designations – six for the three hypostases:¹⁹

Εἰ δὲ δεῖ καὶ Συλλογισμοῖς χρῆσασθαι, πρὸς τοὺς μέγα θαρρόντας ἐπὶ τοῖς οἰκείοις συλλογισμοῖς, οὐχ ἡμετέροις χρησόμεθα εἰς τὴν τοῦ ζητήματος τούτου καθάπαξ εὕρεσιν, ἢ ἀπόδειξιν, ἀλλ’ ὁ ἱερὸς

And if there is a need to use syllogisms (in an argument) against those who are especially bold with the syllogisms of their own [sc., the Latins], we do not use our own syllogisms for finding the resolution or a demonstration for

¹⁸ *Responsio ad syllogismos Marci Ephesi de processu spiritus sancti*, 11b; published among the *dubia* of Gennadios Scholarios: Martin Jugie, Louis Petit, Xenophon A. Siderides, *Oeuvres complètes de Georges (Gennadios) Scholarios*, vol. 3 (Paris: Maison de la bonne presse, 1930) 476–538, (p. 521). For the problems of authorship (Bessarion of Nicaea?), see Marie-Hélène Blanchet, *Georges-Gennadios Scholarios (vers 1400–vers 1472): Un intellectuel orthodoxe face à la disparition de l’Empire Byzantin*. Archives de l’Orient Chrétien, 20 (Paris: Institut français des études byzantines, 2008), p. 480.

¹⁹ Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, τ. Α’, p. 487.

Ἱερόθεος ὁ τοῦ ἐξ Ἀρείου Πάγου Διονυσίου διδάσκαλος, ἀντὶ πάντων ἡμῖν ἐξαρκέσει πρὸς πίστωσιν. Οὗτος γὰρ ἓνα κύκλον ἐξαγωνίζων, καὶ τὰς τούτου γωνίας κέντρα κύκλων ποιούμενος ἕξ, διὰ Συλλογισμῶν ἐξ τὸ παρ’ ἡμῶν ζητούμενον πανσόφως ἀποδείκνυσιν, τιθεὶς ὡς ἐν διαγράμματι. Α. μὲν τὸν Πατέρα. Β. δὲν τὸν Υἱόν. Γ. δὲ τὸ Πνεῦμα τὸ ἅγιον· καὶ τὸν μὲν Προβολέα Ω, τὸν δὲ Λόγον Ψ, τὸ δὲ Πρόβλημα Χ.²⁰

this question, but holy Hierotheos, the teacher of Dionysius from the Areopagus, instead of all others will satisfy for assurance. This one [Hierotheos], inscribing a hexagon into one circle and making from its angular points six centres of circles, demonstrates in an all-wise manner with six syllogisms what we are asking for, putting them as if in a diagram: A for the Father, B for the Son, Γ for the Holy Spirit; and for the Projector Ω, for the Logos Ψ, and for the Projection X.

The six larger circles are not mentioned in the Bryennios’s text, but the chart itself is not authored by Bryennios, being a work of Hierotheos. We will return to them later (section 6).

The six syllogisms are written down in two ways: using the alphabetic designations and in the words, both on the chart and in the text.²¹ In Table 1 below, I am presenting only the “formulae” with alphabetic designations and their translation.

Table 1.

1	τὸ Α τοῦ Β Α. τὸ δὲ Β οὐ καθὸ Β, ἀλλὰ καθὸ Ψ τοῦ τοῦ Γ ἐστὶ. τὸ ἄρα Α, οὐ καθὸ Α, ἀλλὰ καθὸ Ω τοῦ Γ ἐστίν.	A is A of B, but B is of Γ not as B but as Ψ. Therefore, A is of Γ not as A but as Ω.
2	τὸ Β τοῦ Α Β. τὸ δὲ Α οὐ καθὸ Α, ἀλλὰ καθὸ Ω τοῦ τοῦ Γ ἐστὶ. τὸ ἄρα Β, οὐ καθὸ Β, ἀλλὰ καθὸ Ψ τοῦ Γ ἐστίν.	B is B of A, but A is of Γ not as A but as Ω. Therefore, B is of Γ not as B but as Ψ.
3	τὸ Γ τοῦ Ψ Γ. τὸ δὲ Ψ οὐ καθὸ Ψ, ἀλλὰ καθὸ Β τοῦ τοῦ Α ἐστὶ. τὸ ἄρα Γ, οὐ καθὸ Γ, ἀλλὰ καθὸ Χ τοῦ Α ἐστίν.	Γ is Γ of Ψ, but Ψ is of Α not as Ψ but as Β. Therefore, Γ is of Α not as Γ but as Χ.
4	τὸ Ω τοῦ Ψ Ω. τὸ δὲ Ψ οὐ καθὸ Ψ, ἀλλὰ καθὸ Γ τοῦ τοῦ Β ἐστὶ. τὸ ἄρα Ω, οὐ καθὸ Ω, ἀλλὰ καθὸ Α τοῦ Β ἐστίν.	Ω is Ω of Ψ, but Ψ is of Β not as Ψ but as Γ. Therefore, Ω is of Β not as Ω but as Α.
5	τὸ Ψ τοῦ Γ Ψ. τὸ δὲ Γ οὐ καθὸ Γ, ἀλλὰ καθὸ Χ τοῦ τοῦ Α ἐστὶ. τὸ ἄρα Ψ, οὐ καθὸ Ψ, ἀλλὰ καθὸ Β τοῦ Α ἐστίν.	Ψ is Ψ of Γ, but Γ is of Α not as Γ but as Χ. Therefore, Ψ is of Α not as Ψ but as Β.
6	τὸ Χ τοῦ Ω Χ. τὸ δὲ Ω οὐ καθὸ Ω, ἀλλὰ καθὸ Α τοῦ τοῦ Β ἐστὶ. τὸ ἄρα Χ, οὐ καθὸ Χ, ἀλλὰ καθὸ Γ τοῦ Β ἐστίν.	Χ is Χ of Ω, but Ω is of Β not as Ω but as Α. Therefore, Χ is of Β not as Χ but as Γ.

What means “to be of” is explained with a very long but recursive description.²² It would be sufficient to quote it only in a part:²³

ὁ Υἱὸς, τὸ μὲν Υἱὸς ὄνομα τοῦτο, καθὸ Υἱὸς, πρὸς μόνον κέκεται τὸν Πατέρα· Πατὴρ γὰρ ἐστὶν ἑνὸς Υἱὸς μόνου, καὶ οὐ δυοῖν· τὸ δὲ Λόγος ὄνομα, ὃ μόνος ἐν τῇ ὑπερθέῳ Τριάδι πλουτεῖ, οὐ μόνον ἐστὶ τοῦ Πατρὸς καθὸ νοῦ, ἀλλὰ δὴ καὶ τοῦ Πνεύματος, καθ’ ἕτερον τρόπον...

The Son, because he is the one who is the Son, possesses this name of Son *vis-à-vis* the Father alone, for he is the Son of the Father alone, not of the two (hypostases); but the name of Logos which belongs to the Son alone within the super-divine Trinity has reference not only to the Father as the one who is the Intellect, but also to the Spirit in another way...

And, in the same manner, the Spirit is the Spirit of the Father in two different ways: the Spirit is the Projection only *vis-à-vis* the Father as the Projector, that is that who caused him to proceed (this is a causal relation), because the Spirit is not the Son of the Father, but the Spirit is the Spirit of both Father and Son in the sense of non-causal relation, that is, not as the Projection. The Spirit is not the

²⁰ The alphabetic designations by Hierotheos were slightly different – at least, in the only published work where graphical charts are described: αω for Ω, βψ for Ψ, and γχ for Χ; see Hierotheos, *A Homily against the Latins*, ed. Ἰωαννίδης, Ὁ *Τερομόναχος Ἱερόθεος*, 165–233 (pp. 185–6).

²¹ Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, τ. Α’, pp. 491–92.

²² *Ibid.*, pp. 487–89 and 492–97.

²³ *Ibid.*, p. 489.

Projection of the Son or of the Logos, but the Spirit is the Spirit of the Son, as well as the Son is the Son of the Spirit as the Logos.²⁴ And so on.

We will call the three pairs (combinations) “Father and Projector”, “Son and Logos”, and “Spirit and Projection” non-extensional pairs. As we will see, they occur in the middle of other possible extensional pairs. The resulting situation could be compared with an impossible row of natural numbers from 1 to 3, where 2 occurs two times: 1, 2, 2, 3. These two “2” are equal but different in a non-extensional way.

It is easy to show that the six syllogisms from Table 1 are in fact a unique syllogism but written down six times for each of the six permutations by two from three. Let us rewrite these six syllogisms in the following manner:

- x_1 , x_2 , and x_3 – the three hypostases after the mental operation of breaking the symmetry, that is, when one of them is already taken as the first and another one as the second. The symmetry means that all the six possible permutations are implied. Symmetry breaking means that the proposed indexes 1, 2, and 3 belong to the situation that corresponds to a unique specific permutation.
- $\subset$ is the connective “to be x_i of x_j ”, $x_i \subset x_j$, where $1 \leq i \neq j \leq 3$; $\not\subset$ means “not to be x_i of x_j ”.
- $\wedge$ is the sign of conjunction.
- $\therefore$ is the mark of the logical conclusion (“ergo”).
- x_i and $*x_i$ designate the two elements of a non-extensional pair, that is, the two “2” in the row “1, 2, 2, 3” – also after breaking the symmetry. The symmetry which is meant here is that of the border in the paraconsistent topological separation (we will discuss it below).
- x_1 is always the element chosen to be the first in a given ordering (= in a given row of Table 1), x_2 the second, and x_3 the third. Therefore, the first element will be always designated x_1 etc. When choosing between two elements of a non-extensional pair (e.g., A and Ω), x_i designates the element that occurs the first in a given syllogism, whereas $*x_i$ the element that occurs the second.

Each of the six rows of Table 1 after having been rewritten according to these rules results into the following syllogism:

$$(3) \quad \begin{array}{l} x_1 \subset x_2 \wedge x_2 \not\subset x_3 \\ *x_2 \subset x_3 \\ \therefore *x_1 \subset x_3 \wedge x_1 \not\subset x_3 \end{array}$$

The conjunct negative propositions could be omitted as self-evident if we make explicit the following condition implied in the syllogisms by Hierotheos-Bryennios ($\oplus$ means exclusive disjunction):

$$(4) \quad x_i \subset x_j \oplus *x_i \subset x_j$$

In words: in the row of the three elements resulting from symmetry breaking between the six permutations, only one element of each non-extensional pair x_i and $*x_i$ is in a direct non-causal relation with the only one element of any other non-extensional pair. The inter-trinitarian relations are to be discussed with taking into account the non-extensionality.

For the polemical purposes of both Hierotheos and Bryennios, the negative conjuncts in (3) were important, because they explicitly block the *Filioque* (as well as the other isomorphic trinitarian schemes, being therefore a shield against the *reductio ad absurdum* attacks). Our purpose now is different: we need to make explicit Hierotheos’s and Bryennios’s basic logical assumptions, and this is why we are more interested in the explication of the assumption (4) than the negative conjuncts in (3). Taken in the mind the assumption (4), we can simplify the syllogism (3) up to the form (5):

$$(5) \quad \begin{array}{l} x_1 \subset x_2 \\ *x_2 \subset x_3 \end{array}$$

²⁴ *Ibid.*, pp. 489–90.

$$\therefore *x_1 \subset x_3$$

In this form, it becomes obvious that six syllogisms served to establish the idea of a circuit formed with three non-extensional elements, where the element in the middle between any two poles is a non-extensional pair. This is signified by Hierotheos and Bryennios themselves, when they inscribed their hexagon into the circle, and when Bryennios has asked us: ‘Do you see the strange round dance, χορείαν ξένην, that you have never seen before?’²⁵

5. The Non-extensional Pair in the Paraconsistent Topological Separation

As we have seen, a non-extensional pair appears in the middle between the two remaining hypostases. Normally, the Byzantine authors say that, among the three hypostases, there is none in the middle and none at the sides. This means that if we are trying, in our mind, to arrange the three hypostases in any kind of order, we obtain six configurations corresponding to the six ordered pairs possible in a group of three elements. In relation to each of these pairs, the remaining third element is the “middle”. However, unlike the notion of middle treatable in classical logic (or in a row of natural numbers or in any consistent geometry, including the non-Euclidean ones), our specific middle demonstrates a non-extensional behaviour.

The features of the middle between the two paraconsistent poles are known. Let us consider the middle between two sets, A and not- A ($\neg A$), considering this pair of two sets as a subcontrary opposition, that is, an opposition where the classical logic would permit $\neg A \wedge \neg\neg A$ but not $A \wedge \neg A$ (e.g., a hypostasis of the Trinity could be neither Father nor Son but not both Father and Son).

For a comparison with the scheme of Hierotheos, let us consider this case in terms of topology (Fig. 2).²⁶ Therefore, A is a closed set, and $\neg A$ is a closed set as well, the closed complement of A (thus, here $\neg A$ is different from the habitual topological complement of the closed set A , which is an open set). Because A is closed, its border belongs to A exclusively. Because $\neg A$ is closed, its border belongs to $\neg A$ exclusively. Nevertheless, this is the same border. Each point at this border belongs to both A and $\neg A$, and, in both cases, exclusively. For the border, the condition $A \wedge \neg A$ holds.

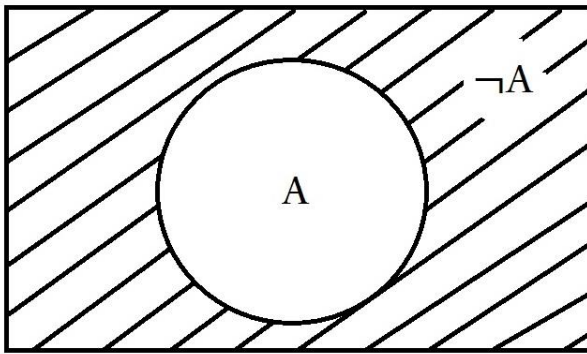


Fig. 2.

The corresponding logic is paraconsistent. According to this logic, for the “third”, or the “middle” hypostasis, hold both identity and difference with each of the two other hypostases. Equivalently, one can say that it is both identical and not identical to itself, thus forming a non-extensional pair.

6. A Venn’s Diagram *avant la lettre*

²⁵ See the quotation in the motto of the present paper. Bryennios (in his *Logos 2 On the Holy Trinity*; Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, τ. Α', p. 24) quotes these words as a part of a fragment ascribed to Maximus the Confessor, but I am sure that the relevant source was late Byzantine; cf. Lourié, ‘Joseph Bryennios’.

²⁶ See, for the details, Chris Mortensen, *Inconsistent Geometry*. Studies in Logic, 27 (London: College Publications, 2010), pp. 5–10.

The larger six circles on Fig. 1 are not discussed in Bryennios's works (at least, the published ones). They were repeated by Bryennios together with the original Hierotheos's drawing and, supposedly, were discussed by Hierotheos somewhere – but not in his published works. Without going any deeper into this subject, I would like to make one observation.

The six circles are overlapping in a way that each of them has intersections with four others but not with the fifth – the one that corresponds to its non-extensionally paired element (e.g., the circle corresponding to “Father” has intersections with all others except the circle corresponding to “Projector” etc.). The overlapping between the two circles corresponding to a non-extensional pair is tangential, that is, covering one dot only – as if without a possibility to be either void or not void. This is a very elegant way of demonstrating a paraconsistent relation.

Other intersections between the circles correspond to both permitted and forbidden combinations of the pairs of hypostases that are considered in the six syllogisms. Were these intersections filled with different ground colours, we would obtain a kind of logical schemes which we call now Venn's diagrams. They were first introduced by John Venn in 1880/1881.²⁷ Hierotheos's “Venn's diagrams” are less user-friendly in respect of colours, but they managed to express non-extensionality.

7. Conclusions

Hieromonk Hierotheos introduced and Joseph Bryennios made “canonical” an explicit language of paraconsistent logic in the Byzantine Triadology. Far from being the first explicitly paraconsistent language, this one was the most formal. It was explained through both a system of syllogisms and the corresponding logical scheme.

The appearance of a logical scheme in the modern sense of word is also something unusual in Byzantium, even though some rudimentary schemes have appeared previously.

Mark of Ephesus in his *epitaphion* to Bryennios called him Ὁ τοῦ Λόγου πρόμαχος ἐνθέοις λόγοις ‘The front-fighter of the Logos with divine *logoi*’.²⁸ I hope that the review of his logical contribution that we have just finished would substantiate the claim that this verse by Mark could be reread as referring as well to the use of logics in our modern sense of word, especially in the sense of coming in response with recent studies in paraconsistent logic.

Joseph Bryennios, however, was not a pioneer of such studies in Byzantium. The major part of his logical achievements was borrowed, without his knowledge, in the legacy of hieromonk Hierotheos. The latter was probably the most innovative logical mind of the late Byzantium, whose non-standard mode of thinking turned out to be almost unbearable for the contemporaries. I hope that the present paper would serve as an introduction to a future study of Hierotheos's theological and logical work.

²⁷ John Venn, *Symbolic Logic* (London: Macmillan and Co., 1881), pp. 100–125 (ch. 5). The substance of this chapter was previously published in a journal in 1880 (cf. *ibid.*, p. VIII).

²⁸ Published in: Ἰωσήφ μοναχοῦ τοῦ Βρυεννίου, *Τὰ παραλειπόμενα*, [ed. by Thomas Mandakases (Kasorianos)] (Leipzig: Βρεϊτκόπφ, 1784], pp. ιζ'-ιη' (p. ιη').